Towards possible origin of binding in non-abelian gauge theories

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Estimate of average energy of the non-abelian dipole in the instanton medium is calculated. This energy for the dipole in colour singlet state escalates linearly with the separation increasing for the point-like sources unscreened. And its 'tension' coefficient develops the magnitude pretty similar to one as inferred from the lattice QCD and other model approaches.

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Recent evolution scenario for the QCD matter being created in ultrarelativistic heavy ion collisions proposes interesting explanation of origin of longstanding 'mixed phase' which was found in lattice calculations of QCD at finite temperatures and, seems, is experimentally observed [1]. A key point to account for the phenomenon in Ref.[1] is to justify the presence of colour Coulomb binding interaction and instanton induced interaction (hence, a lot of bound states available) in rather wide temperature region above the critical temperature. In spite of many intuitive inputs the ideas of this scenario (together with some hints at colour singlet states available everywhere in the coupling space in Ref.[2]) may help in gaining insight on the QCD dynamics. Our major concern here is to answer question whether the origin of those bound states approaching the critical temperature is instructive to understand the nature of ordinary hadrons.

Since the discovery of (anti-)instantons (the classical solutions to the Yang-Mills equations with nontrivial topological features) the problem of the gauge interaction between instantons is under intensive study and the dipole interaction of those pseudo-particles with an external field has already been argued in the pioneering papers [3]. This very important conclusion was grounded on examining the reaction of a single instanton while in a weak slowly varying (constant) external field at distances compared to the instanton scale size. We follow that standard approach in considering the approximate solution of the Yang-Mills equations as

$$\mathcal{A}^{a}_{\mu}(x) = A^{a}_{\mu}(x) + B^{a}_{\mu}(x) . \tag{1}$$

Here the first term denotes the field of a single (anti-)instanton in the singular gauge as we are planning to consider an (anti-)instanton ensemble and need to have the non-trivial topology at the singularity point

$$A^{a}_{\mu}(x) = \frac{2}{g} \,\omega^{ab} \bar{\eta}_{b\mu\nu} \,\frac{\rho^{2}}{y^{2} + \rho^{2}} \,\frac{y_{\nu}}{y^{2}} \,, \tag{2}$$

where ρ is an arbitrary parameter characterizing the instanton size centered at the coordinate z and colour orientation defined by the matrix ω , y = x - z, $\bar{\eta}_{b\mu\nu}$ is the 't Hooft symbol (for antiinstanton $\bar{\eta} \to \eta$) and g is the coupling constant of non-abelian field. The second term describes
an external field. In fact, the result of Ref. [3] as to the instanton interaction with external field
(the CDG term) may be obtained in more general way by calculating the corresponding effective
Lagrangian (without specifying the external field characteristics) [4]. For the sake of simplicity we
limit ourselves here to dealing with SU(2) group only and begin studying an external field originated
by immovable Euclidean colour point-like source $e\delta^{3a}$ and Euclidean colour dipole $\pm e\delta^{3a}$ which is

one of two configurations only providing with the static solutions. Fixing the location of point-like sources is gauge invariant unlike specifying their orientation in 'isotopic' space. In order to get rid of this 'vice' [5] the solution should be characterized by its energy and total 'isospin' ('isospin' of source plus 'isospin' carried by the Yang-Mills field) because these quantities are gauge invariant in addition to being conserved.

With this 'set-up' we could now claim that the results obtained before have maintained the interacting terms proportional e/g only and in this paper we aim to analyze the contributions proportional e^2 which display rather indicative behaviour of asymptotic energy of Euclidean non-abelian point-like sources while in the instanton liquid (IL) [6] and have nothing to do with the additional corrections due to the quantum fluctuations about the saddle point. We treat the potentials in their Euclidean forms and then the following changes of the field and Euclidean point-like source variables are valid $B_0 \to iB_4$, $e \to -ie$ at transition from the Minkowski space. Actually, last variable change is resulted from the corresponding transformations of spinor fields $\psi \to \hat{\psi}$, $\bar{\psi} \to -i\hat{\psi}^{\dagger}$, $\gamma_0 \to \gamma_4$. It means we are in full accordance with electrodynamics where the practical way to have a pithy theory in Euclidean space is to make a transition to an imaginary charge. Surely, these Euclidean sources generate the fields of the same nature as ones we face taking into account gluon field quantum fluctuations a_{qu} around the classical instanton solutions $A = A_{inst} + a_{qu}$.

For Euclidean point-like source and dipole in colourless state we take the Coulomb solutions of the classical Yang-Mills equations in the following forms

$$B^{a}_{\mu}(x) = (\mathbf{0}, \delta^{a3} \varphi), \ \varphi = \frac{e}{4\pi} \frac{1}{|\mathbf{x} - \mathbf{z}_{e}|},$$

$$B^{a}_{\mu}(x) = (\mathbf{0}, \delta^{a3} \varphi), \ \varphi = \frac{e}{4\pi} \left(\frac{1}{|\mathbf{x} - \mathbf{z}_{1}|} - \frac{1}{|\mathbf{x} - \mathbf{z}_{2}|} \right),$$
(3)

respectively, with z_e being a coordinate of point-like source in peace and z_1 , z_2 as coordinates characterizing a dipole. As known, the field originated by particle source in 4d-space develops the cone-like shape with the edge being situated just at the particle creation. Then getting away from that point the field becomes fully developed and steady in the area neighboring to it. In distant area where the field penetrates into the vacuum it should be described by the retarded solution obeying, in particular, the Lorentz gauge $\partial_{\mu}B_{\mu}^{a}=0$ (which is also valid for Eq.(2)). Here we are interested in studying the interactions in the neighboring field area and the cylinder-symmetrical Coulomb field (which is x_4 independent as in Eq.(3)) might be its relevant image in 4d-space.

Thus, the field strength tensor is given as

$$G^{a}_{\mu\nu} = \partial_{\mu}\mathcal{A}^{a}_{\nu} - \partial_{\nu}\mathcal{A}^{a}_{\mu} + g \,\,\varepsilon^{abc}\mathcal{A}^{b}_{\mu}\mathcal{A}^{c}_{\nu} \,\,, \tag{4}$$

where ε^{abc} is the entirely asymmetric tensor and for the field superposition of Eq.(1) it can be given by

$$G^{a}_{\mu\nu} = G^{a}_{\mu\nu}(B) + G^{a}_{\mu\nu}(A) + G^{a}_{\mu\nu}(A, B) ,$$

$$G^{a}_{\mu\nu}(A, B) = g \, \varepsilon^{abc} (B^{b}_{\mu}A^{c}_{\nu} + A^{b}_{\mu}B^{c}_{\nu}) ,$$
(5)

if $G^a_{\mu\nu}(A)$ and $G^a_{\mu\nu}(B)$ are defined as in Eq.(4). Then gluon field strength tensor squared reads

$$G^{a}_{\mu\nu} G^{a}_{\mu\nu} = G^{a}_{\mu\nu}(B) G^{a}_{\mu\nu}(B) + G^{a}_{\mu\nu}(A) G^{a}_{\mu\nu}(A) + G^{a}_{\mu\nu}(A, B) G^{a}_{\mu\nu}(A, B) + 2 G^{a}_{\mu\nu}(B) G^{a}_{\mu\nu}(A) + 2 G^{a}_{\mu\nu}(B) G^{a}_{\mu\nu}(A, B) + 2 G^{a}_{\mu\nu}(A) G^{a}_{\mu\nu}(A, B) .$$

$$(6)$$

The various terms of Eq.(6) provide the contributions of different kinds to the total action of full initial system of point-like sources and pseudo-particle

$$S = \int dx \left(\frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu} + j^a_{\mu} \mathcal{A}^a_{\mu} \right) = S_e(B) + \beta + S_{int} . \tag{7}$$

So, the first and second terms of Eq.(6) account for the source self-energy S_e (for the dipole it should be supplemented by the Coulomb potential of interacting sources) and the single instanton action $\beta = 8\pi^2/g^2$. At small distances the first term is regularized by introducing the source 'size' as in classical electrodynamics. These terms are proportional to e^2 and g^{-2} , respectively and, clearly, out of our interest here. The fourth and last terms of Eq.(6) together with the term $j^a_\mu A^a_\mu$ of Eq.(7) account for the contribution to the interacting part of the action S_{int} proportional to e/g. And the third (repulsive) term of Eq.(6) only leads to the contribution proportional to e^2 as the fifth term is equal zero owing to the gauge choice. Collecting those non-zero contributions we have after performing the calculations the following result

$$S_{int} = -\frac{e}{g} \bar{\eta}_{k4i} \,\omega^{3k} I_i + \left(\frac{e}{4\pi}\right)^2 J + \left(\frac{e}{4\pi}\right)^2 K_{kl} \,\omega^{3k} \omega^{3l} \,. \tag{8}$$

The exact form of I_i (where the CDG term is absorbed) is unnecessary in this paper because going to relate an external field to the stochastic field of fluctuations (the IL model) we have to average over the colour orientation of (anti-)instanton. It leads to disappearance of the dipole contribution and the first nonvanishing correction, as known, comes from a condensate term (but in next order). Two other terms result from the 'mixed' component of field strength tensor which looks like

$$G_{4i}^{a}(A,B) = 2 \frac{e}{4\pi} \varepsilon^{a3c} \omega^{ck} \bar{\eta}_{ki\alpha} \frac{y_{\alpha}}{y^{2}} \frac{\rho^{2}}{(y^{2} + \rho^{2})} \frac{1}{|\mathbf{y} + \mathbf{\Delta}|},$$

$$G_{4i}^{a}(A,B) = 2 \frac{e}{4\pi} \varepsilon^{a3c} \omega^{ck} \bar{\eta}_{ki\alpha} \frac{y_{\alpha}}{y^{2}} \frac{\rho^{2}}{(y^{2} + \rho^{2})} \left(\frac{1}{|\mathbf{y} + \mathbf{\Delta}_{1}|} - \frac{1}{|\mathbf{y} + \mathbf{\Delta}_{2}|} \right),$$

$$(9)$$

for the point-like source and for the field of colourless dipole, respectively, and where $\Delta = z - z_e$, $\Delta_{1,2} = z - z_{1,2}$. The other contributions to $G^a_{\mu\nu}(A,B)$ are absent because of the gauge used. In order to handle the further formulae easily it is practical to introduce new dimensionless coordinates as $x/\rho \to x$. Then for the single source the function J and the tensor K take the following forms

$$J = 2 \int dy \; \frac{2 \; y^2 - \mathbf{y}^2}{y^4 \; (y^2 + 1)^2 \; |\mathbf{y} + \mathbf{\Delta}|^2} \; ,$$

$$K_{kl} = 2 \int dy \, \frac{y_k y_l}{y^4} \, \frac{1}{(y^2 + 1)^2 \, |\boldsymbol{y} + \boldsymbol{\Delta}|^2} \, ,$$

and can not be integrated in the elementary functions. Fortunately, we need to know their asymptotic values at $\Delta \to \infty$ only

$$J \simeq \frac{5\pi^2}{2} \frac{1}{\Delta^2} \ ,$$

and for the components of the 2-nd rank tensor

$$K_{ij} = \delta_{ij} K_1 + \hat{\Delta}_i \hat{\Delta}_j K_2 ,$$

we have

$$K_1 \simeq \frac{\pi^2}{2} \frac{1}{\Lambda^2} \; , \; \; K_2 \simeq 0 \; .$$

Clearly, the 'mixed' component of field strength tensor is of purely non-abelian origin but its contribution to the action of whole system (point-like sources and pseudo-particle) takes the form of self-interacting Euclidean source $\sim e^2$ although it is descended from the instanton field and field generated by sources. It seems, this simple but still amazing fact was not explored properly.

Now we are trying to analyze the pseudo-particle behaviour in the field of Euclidean non-abelian source and develop the perturbative description related to the pseudo-particle itself. Realizing such a program we 'compel' the pseudo-particle parameters to be the functions of 'outside influence', i.e.

putting $\rho \to R(x,z)$, $\omega \to \Omega(x,z)$. These new fields-parameters are calculated within the multipole expansion and then, for example, for the (anti-)instanton size we have

$$R_{in}(x,z) = \rho + c_{\mu} y_{\mu} + c_{\mu\nu} y_{\mu} y_{\nu} + \dots , \qquad |y| \le D$$

$$R_{out}(x,z) = \rho + d_{\mu} \frac{y_{\mu}}{y^{2}} + d_{\mu\nu} \frac{y_{\mu}}{y^{2}} \frac{y_{\nu}}{y^{2}} + \dots , \qquad |y| > D ,$$
(10)

Similar expressions could be written down for the (anti-)instanton orientation in the colour space $\Omega(x,z)$ with D being a certain parameter fixing the radius of sphere where the multipole expansion growing with the distance increasing should be changed for the decreasing one being a result of deformation regularity constraint imposed. Then the coefficients of multipole expansion $c_{\mu}, c_{\mu\nu}, \ldots$ and $d_{\mu}, d_{\mu\nu}, \ldots$ are the functions of external impact. It turns out this approach allows us to trace the evolution of approximate solution for the deformed (crumpled) (anti-)instanton as a function of distance and, moreover, to suggest the self-consistent description of pseudo-particle and point-like source fields within the approximate solution of Eq.(1) (our paper has been completed recently). The role of deformation fields was found to be essential in the local vicinity of source (at distances of order $\leq 2\rho$) and results in considerable growth of the pseudoparticle action.

Proceeding to the calculation of average energy of point-like source immersed into IL we have to remember that one should work with the characteristic configuration which is the superposition of instanton and anti-instanton fields also supplemented by the source field $B^a_{\mu}(x)$, i.e.

$$\mathcal{A}^{a}_{\mu}(x) = B^{a}_{\mu}(x) + \sum_{i=1}^{N} A^{a}_{\mu}(x; \gamma_{i}) , \qquad (11)$$

where $\gamma_i = (\rho_i, z_i, \omega_i)$ are the parameters describing the *i*-th (anti-)instanton. This would-be superposition ansatz is a solution to the Yang-Mills equations only in the limit of infinite separation but approximates well enough the dominant configuration saturating the functional integral [3]. The IL density at large distances from source practically coincides with its asymptotic value $n(\Delta) \sim n_0 e^{\beta - S} \simeq n_0$ because the action of any pseudo-particle here is approximately equal β . The quantity we are interested in should be defined by averaging S over the pseudo-particle positions and their colour orientations (taking all pseudo-particles of the same size at the moment) as

$$\langle S \rangle = \prod_{i=1}^{N} \int \frac{dz_i}{V} \int d\omega_i \quad S =$$

$$= \frac{e^2}{4\pi} \frac{1}{a} X_4 + N \beta + N \int \frac{d\Delta}{L^3} \left(\frac{e}{4\pi}\right)^2 \left(J + \frac{K_{ii}}{3}\right) , \qquad (12)$$

here L is a formal upper integration limit, $V = L^3X_4$ defines the IL volume, X_4 is an upper bound of the 'time' integration, N denotes the total number of pseudo-particles and a is a source 'size' value (on strong interaction scale, of course). Taking the asymptotic functions of J and K and returning to the dimensional variables for the moment one instanton contribution to the average action can be written down in the following form

$$\langle S \rangle \simeq \frac{e^2}{4\pi} \frac{1}{a} X_4 + \frac{N}{V} \beta L^3 X_4 + \frac{N}{V} \frac{6\pi^3}{\beta} \frac{e^2}{g^2} \bar{\rho}^2 L X_4 ,$$
 (13)

where $\bar{\rho}$ is the mean size of (anti-)instanton in IL. The result is given in the form with the common factor X_4 extracted because our concern now is the (anti-)instanton behaviour in the source field background where the field is fully developed and the solution possesses the scaling property at any time slice. In the limit $N, V \to \infty$ and at the IL density n = N/V fixed this result becomes

$$E \simeq \frac{e^2}{4\pi} \frac{1}{a} + n \beta L^3 + n \frac{6\pi^3}{\beta} \frac{e^2}{g^2} \bar{\rho}^2 L$$
 (14)

The last term here looks like a correction to the gluon condensate (the second term). However, this contribution linearly increasing with L and proportional to e^2 has different physical meaning of additional contribution to the source self-energy

$$E \simeq \sigma L , \quad \sigma = n \frac{6\pi^3}{\beta} \frac{e^2}{g^2} \bar{\rho}^2 .$$
 (15)

From the view point of the CDG 'ideology' [3] an appearance of such a term is quite understandable if we remember that for long-range Coulomb solutions Eq.(3) the tensor $G^a_{\mu\nu}$ falls like r^{-2} at large distances (similar to merons and regular instantons), markedly slower than one for the instanton ($\sim r^{-4}$). Besides, the colour 'magnetic field' behaviour for our configuration (of time slice) with point-like source looks as

$$H_k^a = -\delta^{a3} \; \frac{x_k}{|\boldsymbol{x}|^3} \; ,$$

bringing the result to the formal analogy with the Polyakov's model of vacuum at a fixed time albeit its physical meaning here is much more obscure and rather far from the interpretation discussed in [3].

The terms next to leading order could be obtained by a cluster decomposition of the exponential with the interacting term derived, i.e. Eq.(8)

$$\langle \exp(-S_{int}) \rangle_{\omega z} = \exp\left(\sum_{k} \frac{(-1)^k}{k!} \left\langle \left\langle S_{int}^k \right\rangle \right\rangle_{\omega z}\right),$$
 (16)

where the indecies ωz just imply aforementioned averaging over the pseudo-particle positions and their colour orientations and $\langle S_1 \rangle = \langle \langle S_1 \rangle \rangle$, $\langle S_1 S_2 \rangle = \langle S_1 \rangle \langle S_2 \rangle + \langle \langle S_1 S_2 \rangle \rangle$, ... Then, for example, the second order contribution generated by dipole interaction I_i looks like

$$\langle\langle\left(S_{int}^{(I)}\right)^2\rangle\rangle = \frac{e^2}{g^2} \frac{8\pi^3}{3} \left(\pi - \frac{8}{3}\right) \frac{\bar{\rho}^3}{L^3} ,$$

and results in (due to nontrivial dielectic IL properties [3]) the regular correction to the average IL energy (14) as

$$\Delta E^{(I)} = -\frac{N}{V} \frac{e^2}{q^2} \frac{4\pi^3}{3} \left(\pi - \frac{8}{3}\right) \bar{\rho}^3 . \tag{17}$$

Now dealing with the field of colour dipole in the colour singlet state we are able to demonstrate the resembling reaction of IL once more. The contribution to average energy E is defined at large distances by the integral very similar to that for the configuration with one single source

$$I_d = \int d\mathbf{\Delta}_1 \left(\frac{1}{\mathbf{\Delta}_1^2} - 2 \, \frac{1}{|\mathbf{\Delta}_1||\mathbf{\Delta}_2|} + \frac{1}{\mathbf{\Delta}_2^2} \right) \,. \tag{18}$$

When the source separation $l = |z_1 - z_2|$ is going to zero, the field disappears and the final result should be zero. There are only two parameters to operate with the integral, they are L and l. The dimensional analysis reveals that the integral is the linear function of both but the l-dependence only obeys the requirement of integral petering out at $l \to 0$. It is easy to receive the equation

$$\frac{I_d}{4\pi} = L - 2\left(L - \frac{l}{2}\right) + L = l$$

for determining the coefficient (the contributions of three integrals are shown separately here). Finally the average dipole energy for the colour singlet (c.s.) state reads

$$E \simeq \sigma l , \quad (c.s.) ,$$
 (19)

The 'tension' value σ for the IL characteristic parameters $\frac{\bar{\rho}}{R} \simeq \frac{1}{3}$ where \bar{R} is the mean separation of pseudo-particles, $n = \bar{R}^{-4}$, $\beta \simeq 12$, $\bar{\rho} \simeq 1~{\rm GeV^{-1}}$ [6],[7] comes about $\sigma \simeq 0.6~{\rm GeV/fm}$ (if one takes for the brief estimate for the source intensity $e \simeq g$). This result looks fully relevant in view of the qualitative character of estimates which IL is able to provide with. Moreover, such a value is in reasonable agreement with the estimates extracted from the potential models for heavy quarkonia, for example. If one intends to explore the magnitudes like $\langle S_{int} e^{-S_{int}} \rangle$ (which could model an effect of suppressing pseudo-particle contribution in the source vicinity) in numerical calculations it becomes evident the linearly increasing behaviour starts to form at $\Delta/\bar{\rho} \sim 3$ —4. Thus, the asymptotic estimate obtained shows the Euclidean source energy in IL gives the major contribution to the generating functional in quasi-classical approximation if all the coupling constant are frozen at the scale of mean instanton size $\bar{\rho}$.

The linearity of static potential obtained delivers, as we believe one interesting message. The energy growth with the distance increasing hints strongly at a 'great difficulty' of bringing the Euclidean colour source into IL because the source mass (the additional contribution received should be treated just in this way) is unboundedly increasing if the Debye screening effects do not enter the play. It can be shown also that corresponding integral in the color triplet (c.t.) state leads to the result

$$\frac{I_d}{4\pi} = 4 L - l , \qquad (20)$$

with color triplet mean energy

$$E \simeq \sigma \ (4 \ L - l) \ , \quad (c.t.) \ ,$$
 (21)

what clearly demonstrates again it is difficult to have the states with open colour in IL. Besides, it encourages that studying the instanton correlations and collective degrees of freedom induced by them could be the promising way to grasp the full QCD dynamics.

If one strives to go away from the approximation of specifying the source orientation in the 'isotopic' space then the natural generalization of the results obtained takes the form ('sources of large intensity')

$$\frac{I_d}{4\pi} = (\widetilde{P}_1\widetilde{P}_1)L + 2(\widetilde{P}_1\widetilde{P}_2)\left(L - \frac{l}{2}\right) + (\widetilde{P}_2\widetilde{P}_2)L = (\widetilde{P}_1 + \widetilde{P}_2)^2 L - (\widetilde{P}_1\widetilde{P}_2) l,$$

and $e\tilde{P}_1$, $e\tilde{P}_2$ are considered as an intensity of the sources with $\tilde{P}=(P^1,P^2,P^3)$ denoting the vector of the unit normalization $|\tilde{P}|=(P^{\alpha}P^{\alpha})^{1/2}=1$ in colour space (the details of two body problem analysis for arbitrary source orientations in colour space could be found in [8],[9]). Surely, the result demonstrates again that it is very difficult to reveal the states with open colour in IL and besides that there is another small parameter in the problem. Indeed, the deviations of vector-sources from the anti-parallel orientations may not be large.

Another point which is worth of a discussion here concerns the Wilson loop behaviour. The absence of the area law for that in [3] is explained by rapid decay ($\sim \Delta^{-4}$) of the corresponding correlators in IL and inadequate contribution of large size instantons. However, what we are able to estimate now emphasizes an essential impact of the part of average $G_{\mu\nu}^2$ originated by the interference term of colour source field and stochastic (instanton) component when one determines such observables as the interaction energy of the probe (infinitely massive) particles. This quantity developing the linear increase at large distances transforms into the constant gluon condensate while in the stochastic background only. (Even in vacuum the energy required to separate two probe colour charges from each other is finite if there are the light quarks.) Perhaps, the physical picture could be made more transparent if other observables are considered.

The estimate of both configurations contribution to the generating functional (see Ref.[6] for notation)

$$Z = \sum_{N} \frac{1}{N!} \prod_{i=1}^{N} \int (d\gamma_i/\rho_i^5) C_{N_c} \tilde{\beta}^{2N_c} e^{-S} , \qquad (22)$$

can be performed with the variational maximum principle. Then the mean energy of Euclidean sources appears in the exponential factor and may be interpreted as the suppression factor for the states with open colour

$$Z \ge e^{-EX_4} ,$$

here we omitted the condensate contribution together with residual Coulomb component. Calculating the Wilson loop with this weight we find the area law behaviour for the colour dipole in the 'isosinglet' state but have no transpaparent physical picture to take it as a signal of forming the string solution.

We believe the result above shows strong interaction vacuum picture based on the localized lumps of topological charges (which is highly successful in describing many nonperturbative QCD phenomena and hadron phenomenology but at the same time was failling in treating a confinement) still have enough credibility to become a cogent theory of QCD vacuum providing all-round understanding of strong interaction dynamics on the same footing in lieu of a set of individual mechanisms.

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